Math Review For Economics–Solving Equations

Name:

Solve for the Specified Variable

Often an equation contains several variables, and we must express one variable in terms of the others. In the following problems, solve the equation for the specified variable.

Example 1: Solve for y.

$$xy + y - 2x = 1 - 3y$$

To do this, move all terms involving y to one side of the equation and all terms without y to the other.

$xy + y + 3y$	$= 1 + 2x$	Combine like terms.
$xy + 4y$	$= 1 + 2x$	Factor y from the left side.
$y(x + 4)$	$= 1 + 2x$	Divide by the expression next to y.
y	$= \dfrac{1 + 2x}{x + 4}$	

Practice problems: Solve for the specified variable.

1. $T = r + sp$, for p.

2. $F = pqV$, for V.

3. $A = P + Prt$, for P.

4. $M_0 + mY = NX - X$, for M_0; and for m.

5. $B = A(1 + I)$, for I.

6. $S = \dfrac{a_1 - a_1 r}{P - t}$, for a_1.

7. Write R as a function of Y.

   $Y = aR + b$

Solving a System of Equations Using Substitution

Example 2: Solve this system of equations for x.

$$\begin{cases} y - b &= mx \\ y &= nx \end{cases}$$

The second equation is $y = nx$. Substituting this expression for y in the first equation gives us the following equation, which we solve for x:

$$\begin{aligned} nx - b &= mx \\ nx - mx &= b \\ x(n-m) &= b \end{aligned}$$

Thus, $x = \dfrac{b}{n-m}$. Observe that by replacing y using substitution, we obtain an equation with no ys. Substitution is used to eliminate a variable.

Example 3: Solve for C.

$$\begin{cases} Y &= C + D \\ C &= a + bY \end{cases}$$

To decide what substitution to do, look for a variable *besides* C that appears in both equations. Since Y appears in both equations, a good plan of action is to replace Y using substitution.

$$\begin{aligned} C &= a + b(C + D) \\ C &= a + bC + bD \\ C(1-b) &= a + bD \\ C &= \frac{a + bD}{1 - b} \end{aligned}$$

Practice problems.

1. Solve the system in Example 3 for Y.
2. $\begin{cases} wX &= X + N + P \\ N &= N_0 + mX \end{cases}$

   Solve for X.
3. $\begin{cases} Q_d &= a - bP \\ Q_s &= c + dP \\ Q_s &= Q_d \end{cases}$

   Solve for P, Q_s and Q_d.
4. $\begin{cases} T &= S + U \\ S &= a + b(T - W) \\ W &= M + T \\ U &= U_0 + mT \end{cases}$

   Solve for T. (Hint: You'll want to work with the first equation and replace variables besides T. You will need to substitute 3 times.)